

6d)  $\int \frac{2dx}{2\sqrt{x}(1+x)}$   $u=\sqrt{x}$   $\frac{du}{dx} = \frac{x^{-1/2}}{2}$   $Du = \frac{Dx}{2\sqrt{x}}$

$$2 \int \frac{du}{(x+1)} \quad 2 \int \frac{Du}{u^2+1}$$

$$2 \tan^{-1}(u) + C \quad 2 \tan^{-1}(\sqrt{x}) + C$$

sinh(t)  
cosh(t)  
tanh(t)

$$\frac{e^x - e^{-x}}{2} = \sinh(x)$$

$$\frac{e^x + e^{-x}}{2} = \cosh x$$

$$\frac{d}{dx}(\sinh(x)) = \cosh(x)$$

$$\frac{d}{dx}(\cosh(x)) = \sinh(x)$$

Catenary curve



14)  $\int \frac{dx}{2x}$

$u = 2x \quad \frac{du}{dx} = 2$

$\int \frac{du}{2} \quad \int \frac{du}{2u} = \int \frac{1}{2x} \cdot du$

$\frac{1}{2} \int \frac{1}{x} du = \frac{1}{2} \ln(u)$

$\frac{1}{2} \ln|2x| + C$

~~$\frac{1}{2} \ln(2x)$~~

$dx = \frac{du}{2}$

$$46 \int \frac{y \, dy}{\sqrt{y+1}}$$

$$\int \frac{x}{\sqrt{x+1}} \, dx$$

$$\int x(x+1)^{-\frac{1}{2}} \, dx$$

$$u = x+1$$

$$\frac{du}{dx} = 1$$

$$dx = \frac{du}{1}$$

$$\int x(u)^{-\frac{1}{2}} \, dx$$

$$\int x(u)^{-\frac{1}{2}} (du)$$

$$\int (u-1)(u)^{-\frac{1}{2}} (du)$$

$$\int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) (du)$$

$$\left( \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right) + C$$

$$= \left( \frac{2}{3} (x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} \right) + C$$

$$\int \frac{x}{(4x^2+1)^3} dx$$

$$u = 4x^2 + 1$$
$$\frac{du}{dx} = 8x \quad dx = \frac{du}{8x}$$

$$\int \frac{x}{u^3} \cdot \frac{du}{8x} \quad \int \frac{x du}{u^3 8x} \quad \int \frac{du(u)^{-3}}{8} = -\frac{u^{-2}}{16}$$

$$= -\frac{(4x^2+1)^{-2}}{16} + C$$

$$39) \int \sec^3 2x \tan 2x dx$$

$$\frac{1}{6} \int \sec u \tan u \cdot 2 dx$$

$$\frac{1}{6} \sec(u)$$

$$\frac{1}{6} \int 3 \sec^3(u)$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

$$\frac{1}{6} \sec 2x + C$$

$$\int \frac{dx}{\sqrt{1-4x^2}}$$